Stabilized Bordered Block Diagonal Form for Solving Nonlinear Magnetic Field Problems

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For analyzing large-scale engineering problems using numerical methods, parallel computation is an efficient approach. Graph partitioning algorithms in the numerical parallel computation have important theoretical significance and practical applications. This paper presents a stabilized bordered block diagonal form (SBBDF) of Jacobian matrix in an improved Newton-Raphson (N-R) method for solving the problems of nonlinear electromagnetic field. This algorithm can reduce the computing time in parallel computation. It bases on graph partitioning algorithm, and is quite competent for large scale parallel computation for possible reduced iteration steps, which has been demonstrated by numerical experimental results.

Index Terms—**Graph partitioning algorithm, nonlinear magnetic problems, parallel computation, stabilized bordered block diagonal form.**

I. Introduction

Finite element method (FEM) is usually used as a numerical approach for solving elliptic partial differential equations, particularly suitable for problems with complex geometries. FEM is also an effective tool to solve problems of nonlinear magnetic field [1]. In electromagnetic field simulation, the nonlinear equations can be linearized into linear forms by using Newton's method. The popular Newton-Raphson (N-R) method has quadratic rate of convergence. When applying the N-R method to solving nonlinear problems, it is necessary to update the Jacobian matrix and solve the matrix equation in each nonlinear iteration. This could be time consuming when solving problems with large amount of unknowns [2].

With the emergence of the first parallel computer in 1972, a series of types of the array and vector machines have been invented. In the structure of distributed storage system, the messaging time between the processors is related to the message length and the distance between processors [3]. Compared with serial computing, parallel computing can greatly shorten the computing time.

In order to better solve the problem of load balancing in parallel computing, it is necessary to use mathematical language giving the definition of the problem. The mainstream of the existing methods is usually based on task graph [4]. Graph partitioning problem has been applied in many areas, including scientific computing, information systems, operation research and so on [5]. One method of solving equations of large systems, called partitioning, has been investigated in a number of applications [6]. One of the most proper dividing method of parallel processing is making the original graph into the matrix with a stabilized bordered block diagonal form (SBBDF). Generally, in Newton-Raphson (N-R) method for nonlinear electromagnetic problems, the Jacobi matrix between two adjacent iteration steps usually is a sparse matrix. The characteristics are reflected at the partly change of tangent matrix. The graph partition is a dividing and conquering algorithm which prevents local change from wide spreading [7].

In this paper a stabilized bordered block diagonal form (SBBDF) is presented to improve the N-R method for nonlinear magnetic problems. This SBBDF N-R method is based on the graph partitioning of Jacobian matrix and parallel computing. It elevates N-R method and makes it more efficient. A numerical example is presented for analyzing the performance of the employed algorithm.

II. Stabilized Bordered Block Diagonal Form

In this report, the algorithm employed in the N-R method should be ordered into SBBDF [8]. This is a two-step ordering work, first order the matrix M into BBDF as shown in Fig. 2, then order it into SBBDF. The entire ordering process needs to be worked out before the first iteration of N-R method.

$$\begin{bmatrix} D_{11} & & D_{1S} \\ & D_{22} & D_{2S} \\ D_{21} & D_{S2} & D_{SS} \end{bmatrix}$$

Fig. 2. A bordered block diagonal form (BBDF).

The first step of ordering:

Now suppose that S is an attachment set in the row graph of a matrix M. Once S and the attendant edges have been deleted, let $R_1, \ldots, R_m$ are the subsets of the set of rows of M. These rows correspond to the m components of the row graph. Then each column of M has nonzero entries in rows from at most one R.

The second step of ordering:

At the place of the entries the coupling rows are split, a rectangular matrix is constructed. In this case, each new row has nonzero entries in only one nonborder block. Then by appending new columns, a square matrix is produced. This can help to ensure the stretched matrix is structurally non-singular. We apply a strategy called “row stretching” to associate with BBDF matrix to a larger square matrix with SBBD form.

III. SBBDF Newton-Raphson Method

In electromagnetic field, when solving nonlinear problems using FEM, we can obtain a matrix equation

$$J\Delta x = f \qquad (1)$$

where Δx is the unknown increment to be solved. J is the $n \times n$ sparse and symmetric Jacobian matrix. f is the column vector associated with excitations.

Based on the difference of magnetic reluctivity, the Jacobian matrix is permutated into the SBBDF.

$$J=\begin{bmatrix} J_{11} & & & & J_{1n} \\ & J_{22} & & & J_{2n} \\ & & \ddots & & \vdots \\ & & & J_{pp} & J_{pn} \\ J_{n1} & J_{n2} & \cdots & J_{np} & J_{nn} \end{bmatrix} \quad (2)$$

The solving process based on SBBDF has the following steps.

Step 1. Since in nonlinear electromagnetic problems, only part of sparse matrix J is changing, then J can be generated into the SBBDF of (2). Take (2) into (1) , then the system of (1) becomes:

$$\begin{bmatrix} J_{11} & & & & J_{1n} \\ & J_{22} & & & J_{2n} \\ & & \ddots & & \vdots \\ & & & J_{pp} & J_{pn} \\ J_{n1} & J_{n2} & \cdots & J_{np} & J_{nn} \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \vdots \\ \Delta x_p \\ \Delta x_n \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_p \\ f_n \end{bmatrix} \quad (3)$$

Step 2. The equations (8) can also be formed as:

$$\begin{cases} J_{11}\Delta x_1 + J_{1n}\Delta x_n = f_1 \\ J_{22}\Delta x_2 + J_{2n}\Delta x_n = f_2 \\ \vdots \\ J_{pp}\Delta x_p + J_{pn}\Delta x_n = f_p \\ J_{n1}\Delta x_1 + J_{n2}\Delta x_2 + \cdots J_{np}\Delta x_p + J_{nn}\Delta x_n = f_n \end{cases} \quad (4)$$

Eliminating $\Delta x_i, i = 1, 2, \cdots p$ in (9) can obtain

$$J_s \Delta x_n = f_s \quad (5)$$

$$J_s = J_{nn} - \sum_{i=1}^{p} J_{ni} J_{ii}^{-1} J_{in} \quad (6)$$

$$f_s = f_n - \sum_{i=1}^{p} J_{ni} J_{ii}^{-1} f_i \quad (7)$$

$$\Delta x_i = J_{ii}^{-1} f_i - J_{ii}^{-1} J_{in} \Delta x_n, (i = 1,2,\ldots,n-1) \quad (8)$$

Step 3. The following formulas are the foundation of the parallel computing.

$$\begin{gathered} \Delta x_1 = J_{11}^{-1} f_1 - J_{11}^{-1} J_{1n} \Delta x_n \\ \vdots \\ \Delta x_p = J_{pp}^{-1} f_p - J_{pp}^{-1} J_{pn} \Delta x_n \end{gathered} \quad (9)$$

$$A_i = J_{ni} J_{ii}^{-1} J_{in} \quad (10)$$

$$B_i = J_{ii}^{-1} f_i \quad (11)$$

After solving Δx_n from (10), then apply parallel computing to (9), we can obtain Δx_1, $\Delta x_2, \cdots \Delta x_p$.

When solving (5), since the coefficient matrix J_S is a symmetric but not sparse matrix, the calculation consuming still occupies a large amount of computing time [9]. The computing load in one iteration for Δx_n, *A* and *B* is shown in Table I.

TABLE I

CALCULATION AMOUNTS IN ONE ITERATION OF SBBDF N-R METHOD

Items	Formula	Computing load
Δx_n	$J_s^{-1} f_s$	$(n^3+3n^2-n)/3$
A	$J_{ii}^{-1} J_{in}$	$max(n_i)^{1.5}$
B	$J_{ii}^{-1} f_i$	$max(n_i)^{1.5}$

IV. NUMERICAL EXAMPLE

In order to test the efficiency of the SBBDF N-R method in solving magnetic problems, an U-shape electromagnet as shown Fig. 3 is taken as an example. The result of using the N-R method based on traditional N-R method is shown in Fig. 3. The comparison of calculation amount of different part of SBBDF N-R method is shown in Table II. The comparison of total calculation amount of different part is shown in Table III.

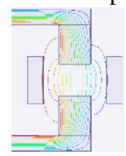

Fig. 3. Flux lines of U-shape electromagnet.

TABLE II

CALCULATION AMOUNTS IN ONE ITERATION

Items	Formula	Computing load
Δx_n	1	206
A	1	61
B	1	53

TABLE III

CALCULATION AMOUNTS WITH DIFFERENT NUMBER OF BLOCKS

	n=2			n=4		
Items	Δx_n	A	B	Δx_n	A	B
computing time (ms)	1278	432	366	729	324	261

V. CONCLUSION

Compared to traditional N-R method, the SBBDF N-R method is able to reduce the computation load in each iteration step. According to the pre process of ordering the Jacobian matrix, the meshing and coding method can affect the computing load. With better mesh, the method can save more computing time and be more efficient.

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